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A SIMILAR PROFILE REGIME IN THE VERTICAL FULLY DEVELOPED FLOW OF GAS–SOLIDS SUSPENSIONS

Y. MOLODTSOF[1]† and D. W. MUZYKA[2]‡
[1]Département de Génie Chimique, Université de Technologie de Compiègne, B.P. 649, 60200 Compiègne, France
[2]University of Western Ontario, London, Ontario, Canada

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Abstract—This paper examines the asymptotic behaviour of general probabilistic multiphase flow equations in fully developed gas–solids suspension flows as the solids concentration tends toward zero. This analysis suggests the existence of self-similar concentration and solids velocity profiles as solutions to general multiphase flow equations at low solids loading. The existing experimental evidence confirms this theoretical result and strongly suggests this to be a general feature of dilute suspension flows. The analysis further suggests that deviations from a similar profile regime as the solids loading is increased can be deduced from the relationship between solids flowrate and mean solids concentrations. Finally, the flow equation analysis provides a partial explanation for the difficulty associated with developing generalizable suspension pressure drop correlations.

Key Words: gas–solids suspensions, vertical flow, dilute phase flow, similar profiles

INTRODUCTION

Gas–solids suspensions flow vertically in many industrial processes, ranging from pneumatic transport to fluidized bed reactors, including risers in the petroleum industry, transport reactors and circulating fluidized beds. For these industrial applications, the properties of the suspension velocity and concentration profiles are at least as important as the overall characterization of the hydrodynamics. However, compared with the considerable number of papers dealing with gas–solids suspensions in general, the literature is rather poor as far as the radial distribution of particles in the flow, or the velocity profiles of both phases are concerned. In addition, experimental studies devoted to the determination of flow patterns are limited to specific operating conditions and their conclusions can rarely be generalized due to the large number of overall variables characterizing the flow of gas–solids mixtures. Moreover, modelling of gas–solids suspension flows is difficult, since the laws governing interphase interactions are far from well understood.

The general probabilistic multiphase flow equations (Molodtsof & Muzyka 1989) provide a rigorous basis for the analysis of the flow structure of suspensions. The work presented in this paper starts from this basis and derives self-similar concentration and velocity profiles as asymptotic solutions to the general equations for vertical gas–solids suspensions in fully developed dilute phase flow conditions.

SURVEY OF THE LITERATURE: EXPERIMENTAL OBSERVATIONS

Several authors have reported radial particle velocity profile measurements (van Zoonen 1962; Konno & Saito 1969; Reddy & Pei 1969; Kramer 1970; Arundel *et al.* 1970, 1974; Lee & Durst 1982; Birchenough & Mason 1976; Maeda *et al.* 1980; Tsuji *et al.* 1984). Kramer (1970), who reports the most complete collection of data, observed that, at constant gas superficial velocity, the velocity profiles of 62 μm glass beads deform slightly when the loading ratio increases, whereas they remain unchanged for 200 μm beads (figures 1 and 2). Solids velocity profiles, essentially independent of solids loading, can also be observed in the results reported by Birchenough & Mason (1976), Maeda *et al.* (1980) and Tsuji *et al.* (1984).

†To whom correspondence should be addressed.
‡Present address: DuPont of Canada, Research Centre, Kingston, Ontario, Canada.

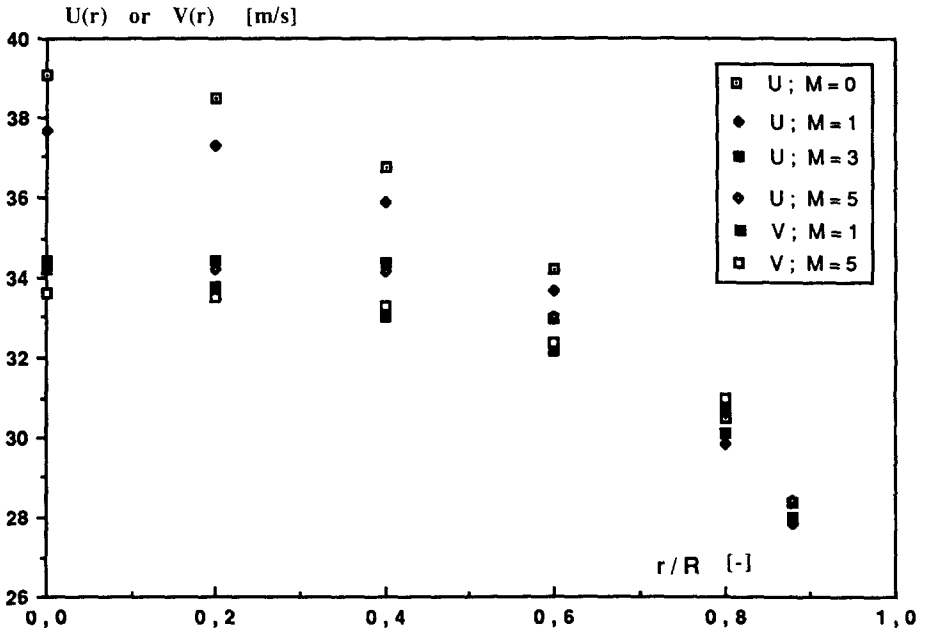

Figure 1. Air and particle velocity profiles reported by Kramer (1970) for 62 μm glass beads carried by air in a 0.5" (12.7 mm) i.d. pipe for a gas Reynolds number of 24,500 and different values of the loading ratio ($M = W_s/W_f$).

On the other hand, gas velocity profiles deform when the particle concentration is increased (Doig & Roper 1967; Kramer 1970; Maeda *et al.* 1980; Tsuji *et al.* 1984): the profile flattens with increasing loading (figures 1 and 2) and becomes even concave in the core of the pipe. The relative velocity between the gas and solid particles is neither equal to the terminal free-fall velocity, nor constant over the cross-section of the pipe; negative relative velocities have even been reported in the vicinity of the wall (Lee & Durst 1982). The latter results from the fact that the velocity of the particles is not necessarily zero at the wall but can be positive or negative depending upon

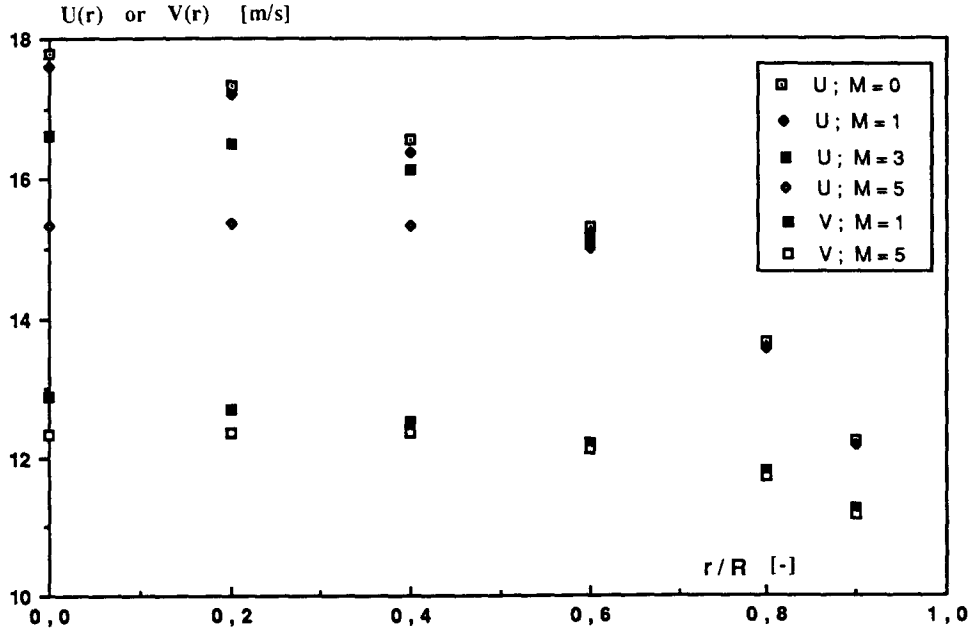

Figure 2. Air and particle velocity profiles reported by Kramer (1970) for 200 μm glass beads carried by air in a 1" (25.4 mm) i.d. pipe for a gas Reynolds number of 24,500 and different values of the loading ratio ($M = W_s/W_f$).

operating conditions. Finally, the presence of the particles modifies the turbulence intensity of the gas flow (Maeda *et al.* 1980; Tsuji *et al.* 1984).

Particle concentration profiles have also been reported. They result mostly from direct measurements (e.g. van Zoonen 1962; Konno & Saito 1969; Weinstein *et al.* 1986; Hartge *et al.* 1986, 1988). Kramer (1970) deduced the particle concentration profiles from particle velocity and number flux measurements. His numerous results show that particle concentrations near the wall can be either greater than or less than the concentration in the core, depending on the operating conditions, although all other authors reported only concave profiles. At constant superficial velocity, the *shape* of the profiles seems to be essentially independent of solids loading. On the effect of particle size and their radial distribution, a recent paper (Dry 1987) reports slight differences between the core and the annulus, in a circulating fluidized bed riser, under special operating conditions.

Some authors tried to combine measurements and modelling for the determination of flow patterns in circulating fluidized bed risers (Rhodes *et al.* 1988; Hartge *et al.* 1988). Their model equations, however, contain too many simplifying assumptions to be generalizable. Moreover, the results obtained by such methods are necessarily limited to operating conditions under which the measurements have been performed.

Despite the relatively large differences in operating conditions (e.g. pipe diameter and nature, particle size and properties, superficial gas velocity range) some common aspects can be traced in the work cited above, at least, under conditions of *constant superficial gas velocity*:

- The solids velocity profiles seem to be independent of average particle concentration.
- The *shape* of particle concentration profiles seems to be independent of solids loading.

In the next two sections, a theoretical basis will be derived allowing the determination of the conditions in which these suggested properties can be generalized.

FULLY DEVELOPED FLOW

General Equations

Molodtsof & Muzyka (1989) developed a general theoretical framework allowing a rigorous mathematical description of multiphase mixtures, termed *probabilistic Eulerian description*. According to this approach, the presence and all physical quantities (i.e. immediate Eulerian variables) of each phase are random variables governed by laws of probability determined by the overall boundary conditions imposed to the flow. A given phase p in the mixture is then characterized by a phase presence probability ϵ_p and probabilistic mean Eulerian variables of the phases are defined as the expected value of the random immediate Eulerian variable under consideration (see the appendix). The general probabilistic multiphase flow equations are written in terms of these "phase mean variables" which are shown to be identifiable with measurable quantities in the flow.

The probabilistic approach to general equations is well-adapted to the analysis of the asymptotic behaviour of gas–solids suspensions to be carried out below since these are *local* equations written in terms of *local* variables defined so as not to be dependent on the existence of an appropriate spatial or temporal averaging domain. Besides, among rigorously derived general flow equations, they are the only ones to account for the specifics of gas–solids suspension flow, particularly addressing the interactions between particles (Muzyka 1985). In the analysis below, all particle phases will be treated as the overall "solids" phase. The conditions under which this is valid (i.e. when all the particles have the same physical properties such as density, regardless of their size or shape) were studied in an earlier paper (Molodtsof & Muzyka 1989).

For any phase p of the mixture, a continuity equation is derived:

$$\frac{\partial}{\partial t}(\rho_p \epsilon_p) + \frac{\partial}{\partial x_j}(\rho_p \epsilon_p V_j^p) = 0, \qquad [1]$$

where ρ_p denotes the phase p density and V_j^p are the components of the phase mean velocity. The momentum equation of the overall "solids" phase, when projected on the Ox_i axis, takes the following form

$$\frac{\partial}{\partial t}(\rho_s \epsilon_s V_i) + \frac{\partial}{\partial x_j}[\rho_s \epsilon_s (V_i V_j + \beta_{ij})] = \rho_s \epsilon_s g_i + F_i + \frac{\partial}{\partial x_j}(\epsilon_s \sigma_{ij}) + \frac{\partial}{\partial x_j}(s_{ij}), \quad [2]$$

where β_{ij} denotes the components of the velocity cofluctuation tensor, g_i stands for the components of the acceleration of gravity and F_i accounts for the gas–solids interaction force. Two stress tensors acting within the particles appear in this equation: σ_{ij} accounts for the stresses linked by the fluid; while s_{ij} is the probabilistic mean of intermittent stresses due to interparticle and/or wall–particle collisions. The momentum equation for the fluid phase is

$$\frac{\partial}{\partial t}(\rho_f \epsilon_f U_i) + \frac{\partial}{\partial x_j}[\rho_f \epsilon_f (U_i U_j + B_{ij})] = \rho_f \epsilon_f g_i - F_i - \frac{\partial}{\partial x_i}(\epsilon_f p) + \frac{\partial}{\partial x_j}(\epsilon_f \tau_{ij}), \quad [3]$$

where U_i and B_{ij} denote, respectively, the components of gas velocity and the velocity cofluctuation tensor, p stands for the pressure and τ_{ij} represents the components of viscous stresses acting within the fluid. In [2] and [3], the l.h.s. represents the inertial terms, while all the external forces acting on the phase are regrouped on the r.h.s.

An additional equation relates gas and solids presence probabilities:

$$\epsilon_s + \epsilon_f = 1. \quad [4]$$

These equations cannot be solved in general, since there is an obvious closure problem associated with them.

Fully developed vertical upflow of a suspension

When a single-phase fluid flows, at a constant rate, in a straight pipe, the longitudinal pressure profile becomes linear beyond a certain distance downstream from the entrance; this situation corresponds to the establishment of flow patterns which are identical in each cross-section of the pipe, and is called fully developed flow. Many authors have intuitively extended this concept to suspension flows. Molodtsof (1985) proposed a rigorous definition for the fully developed flow of a suspension: when a gas–solids mixture flows in the steady-state conditions (i.e. with time-independent overall boundary conditions such as constant flowrates, end pressures etc.) in a straight pipe, the flow is said to be fully developed when the laws of probability governing the presence and the velocity of each phase become independent of the axial coordinate. According to this definition, in [1]–[4] ϵ_s, ϵ_f, U and V, as well as the velocity cofluctuation tensors are independent of the longitudinal coordinate, in fully developed flow conditions. Moreover, the viscous stresses which are determined by the local random velocity fields of the fluid, are functions of the transverse coordinates only. The same holds true for the s_{ij} tensor, since momentum transfers during collisions are determined by the random presence and velocities of particles.

In the case of a vertical pipe of circular cross-section, the flow is axisymmetric, and the only remaining transverse coordinate is the radial distance r to the pipe axis. If x denotes the axial coordinate upwardly directed, the continuity equation [1] for any phase p simplifies as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}(r \rho_p \epsilon_p V_r^p) + \frac{\partial}{\partial x}(\rho_p \epsilon_p V_x^p) = 0. \quad [5]$$

Under fully developed flow conditions, the phase presence and velocities are independent of x; [5] can, therefore, be immediately integrated in the form

$$r \rho_p \epsilon_p V_r^p = \text{const} = [r \rho_p \epsilon_p V_r^p]_{r=R}, \quad [6]$$

where R denotes the pipe radius. In [6], the constant is computed at the wall. It is zero, for $p = s$, since, the probability of presence of the solids is zero at the wall (Molodtsof & Muzyka 1989). The constant is also zero for the fluid ($p = f$), according to the no-slip condition. Therefore, the radial projections of both phases velocities vanish identically throughout a fully developed flow.

The axial projection of [3] takes then the following form:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\rho_f\epsilon_f B_{rx}) = -\rho_f\epsilon_f g - F_x - \epsilon_f\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\epsilon_f\tau_{rx}). \quad [7]$$

Since the gas–solids interaction term F_x is determined by the random flow field of the fluid about the particles, this term is independent of x following the reasoning applied to the other terms reviewed above. The longitudinal pressure gradient is, therefore, independent of the axial coordinate in fully developed flow. Molodtsof (1985) showed that this term is also independent of r. The pressure gradient is, thus, constant throughout the pipe as in fully developed single-phase flow.

Under the same conditions, [2] simplifies as follows:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\rho_s\epsilon_s\beta_{rx}) = -\rho_s\epsilon_s g + F_x + \epsilon_s\frac{\partial}{\partial x}(\sigma_{xx}) + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\epsilon_s\sigma_{rx}) + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rs_{rx}). \quad [8]$$

Molodtsof (1985) showed that σ_{rx} is a function of r only and, that the axial gradient of σ_{xx} is constant throughout the flow and equal, with the opposite sign, to the pressure gradient:

$$\frac{\partial}{\partial x}(\sigma_{xx}) = -\frac{\partial p}{\partial x} = \text{const.} \quad [9]$$

Thus, under fully developed flow conditions, [8] is entirely independent of x and takes the following form:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\rho_s\epsilon_s\beta_{rx}) = -\rho_s\epsilon_s g + F_x - \epsilon_s\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\epsilon_s\sigma_{rx}) + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rs_{rx}). \quad [10]$$

Equations [7] and [10], together with [4], are the general equations of fully developed vertical upflows of a fluid–solids suspension. In the next section, they will be used in the analysis of dilute phase flow.

DILUTE PHASE FLOW OF A SUSPENSION

Reference flow

In the probabilistic Eulerian approach, the phase presence probability ϵ_p of a phase p is equal to the mean local volume fraction occupied by that phase (Molodtsof 1985). Consequently, ϵ_s is the local solids volumetric concentration. An average concentration can be defined as follows:

$$c = \frac{1}{\pi R^2}\int_0^R 2\pi r\epsilon_s(r)\,\mathrm{d}r, \quad [11]$$

which characterizes the average volumetric concentration of solids over a cross-section.

Many of the authors dealing with gas–solids suspensions compare the behaviour of the suspension to that of the gas alone. This *reference flow* is defined as that of the gas alone flowing in the same pipe with the same superficial velocity $\mathscr{U}$ (i.e. the same flowrate) as in the suspension. The momentum equation of this reference flow follows from the equations of single-phase flow:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\rho_f B^0_{rx}) = -\rho_f g - \frac{\partial p^0}{\partial x} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\tau^0_{rx}), \quad [12]$$

where the 0 superscript indicates the reference flow variables. The velocity cofluctuation tensor (multiplied by density) corresponds, then, to the turbulent Reynolds stresses.

When, in pneumatic transport for instance, the solids rate fed into the gas is progressively reduced to zero while the superficial gas velocity $\mathscr{U}$ is maintained constant, the installation becomes empty of solids, i.e. the average solids concentration c reduces progressively to zero and the gas flows alone (reference flow) in the pipe. Therefore, when $c \to 0$, each term in [7] should tend toward the corresponding term of [12]. In other words, [12] is the limit, by continuity, of [7] when $c \to 0$. All fluid phase variables should, then, tend asymptotically toward the corresponding variables of the reference flow as, according to [4], $\epsilon_f \to 1$ throughout the pipe. Therefore, it can be reasonably assumed that as $c \to 0$, $\epsilon_s \to 0$ throughout the pipe in such a way that ϵ_s remains of the same order

of magnitude as c. This hypothesis means that no solids accumulation region exists in a cross-section.

Asymptotic behaviour of gas flow variables

According to the continuity limit assumption stated above, for $c = 0$, the shearing component B_{rx} of the velocity cofluctuation tensor is equal to the corresponding term of the reference flow. In the suspension, this term differs from the latter because of the perturbations induced in the gas flow due to the presence of particles. In the vicinity of a particle, the turbulent cofluctuation B^0_{rx} becomes then, b_{rx}. In the case of a single particle, $b_{rx} \to B^0_{rx}$ beyond distances about several particle diameters. Thus, the volume in which b_{rx} differs from its limiting value is proportional to the volume of the particle. Let k be the ratio of this volume in which the reference flow field is perturbated, to that of the particle. Let us consider now a suspension containing only a few particles such that $k\epsilon_s < \epsilon_f$. This condition corresponds essentially to the non-overlapping of the individual perturbated volumes and certainly becomes satisfied as $c \to 0$. At a given instant, the probability, for a point occupied by the gas, to be located within the perturbated volume is $(k\epsilon_s)$ while the complementary probability is $(1 - k\epsilon_s)$. Thus, if b_{rx} denotes the probabilistic mean cofluctuation in the perturbated volume, one has, on average:

$$B_{rx} = k\epsilon_s b_{rx} + (1 - k\epsilon_s)B^0_{rx}. \qquad [13]$$

Using [4], the following asymptotic form can, finally, be derived:

$$\epsilon_f B_{rx} = B^0_{rx} + \epsilon_s B^1_{rx} + O(c^2), \qquad [14]$$

where, B^1_{rx} is independent of the solids concentration.

An analogous asymptotic expression identifiable with a Maclaurin series development limited to first-order terms can be derived for the other terms of [7]. Indeed, for the viscous term, Molodtsof (1985) presented the following equation (see [A.10] in the appendix):

$$\epsilon_f \tau_{rx} = \mu\left[\frac{d}{dr}(\epsilon_f U_x) + V_1 \frac{d\epsilon_s}{dr}\right], \qquad [15]$$

where V_1 is the "interfacial" mean velocity of the particles. Using a Maclaurin series development for $\epsilon_f U_x$,

$$\epsilon_f U_x = U^0_x + \epsilon_s U^1_x + O(c^2), \qquad [16]$$

[15] leads to

$$\epsilon_f \tau_{rx} = \tau^0_{rx} + \mu\left[\frac{d}{dr}(\epsilon_s U^1_x) + V_1 \frac{d\epsilon_s}{dr}\right] + O(c^2). \qquad [17]$$

The gas–solids interaction force F_x should identically vanish as $c \to 0$. Its asymptotic behaviour is, therefore, of the form:

$$F_x = \epsilon_s F^1_x + O(c^2). \qquad [18]$$

The asymptotic form of the pressure drop term in [7] can be deduced from the numerous experimental results reported in the literature. A general concensus exists (e.g. Muzyka 1985) about the following form in dilute phase flow:

$$\frac{\partial p}{\partial x} = \frac{\partial p^0}{\partial x} + c\frac{\partial p^1}{\partial x}, \qquad [19]$$

where the first term on the r.h.s. is the pressure drop of the gas alone (reference flow), and the coefficient of c in the second term is determined only by the superficial gas velocity for a given solid and a given pipe.

The expressions obtained in [14] and [17]–[19] can now be substituted for the terms appearing in [7]. The reference flow terms are eliminated by subtracting [12]. Neglecting the second-order terms $O(c^2)$, the following result is then obtained:

$$\frac{1}{r}\frac{d}{dr}(r\rho_f \epsilon_s B^1_{rx}) = \epsilon_s \rho_f g - \epsilon_s F^1_x + \left(\epsilon_s \frac{\partial p^0}{\partial x} - c\frac{\partial p^1}{\partial x}\right) + \frac{1}{r}\frac{d}{dr}\left[r\mu\frac{d}{dr}(\epsilon_s U^1_x) + rV_1\frac{d\epsilon_s}{dx}\right]. \qquad [20]$$

When considered in terms of local solids concentration ϵ_s, [20] is a second-order differential equation with variable coefficients, of the following form:

$$A(r)\frac{d^2\epsilon_s}{dr^2} + B(r)\frac{d\epsilon_s}{dr} + C(r)\epsilon_s(r) + cD(r) = 0. \quad [21]$$

It should be noted that if coefficients A, B, C and D do not indirectly depend on either ϵ_s or c, [21] is a homogenous equation in terms of solids concentration.

SELF-SIMILAR PROFILES

Equation [20] (or formally [21]) is, therefore, a differential equation governing the distribution of the particles in a cross-section. If one can compute the coefficients A–D, the complete solution can be obtained by solving this equation with the following boundary conditions (Molodtsof & Muzyka 1989):

$$\epsilon_s = 0 \quad \text{and} \quad \frac{d\epsilon_s}{dr} = 0 \quad \text{for } r = R. \quad [22]$$

These boundary conditions are independent of the average solids concentration c. Thus, the *homogenous* differential equation [21], together with the above boundary conditions *determine the shape* of the solids concentration profile $\epsilon_s(r)$ *regardless of the average concentration* c. In other words, if we define a "reduced" concentration profile:

$$f(r) = \frac{\epsilon_s(r)}{c}, \quad [23]$$

which characterizes the shape of the concentration profile, [21] and [22] divided by c become the equations determining $f(r)$. These equations are independent of c provided that coefficient functions A–D are independent of c. The concentration profiles $\epsilon_s(r)$ would be, then, *self-similar profiles* proportional to the average solids concentration c.

The average concentration c would, then, be determined by the overall solids rate W_s according to the following defining equation:

$$W_s = \int_0^R 2\pi r\epsilon_s(r)V_x(r)\,dr = c\int_0^R 2\pi r f(r)V_x(r)\,dr. \quad [24]$$

The coefficients A–D depend upon the superficial gas velocity $\mathcal{U}$. They are explicitly independent of c as seen in the Maclaurin series developments derived above; but they can be implicit functions of the average concentration if the solids velocity profile depend on c. Indeed, some of the first-order asymptotic terms such as, V_1, the gas–solids interaction term and the additional pressure drop (Muzyka *et al.* 1983) depend on the random particle velocities. Consequently, *solids velocity profiles independent of c are a necessary condition for self-similar concentration profiles.*

In fact, considerable experimental evidence exists which suggests that this necessary condition is satisfied in dilute phase flow. Considering, however, only theoretical arguments, it can be noted that the solids velocity field should be determined by [10] in which all the terms should identically vanish as $c \to 0$. No zeroth-order term exist therefore in the asymptotic form of this equation. A first-order approximation can be derived using Maclaurin series developments as follows:

$$\epsilon_s\psi_s = \epsilon_s\psi_s^1 + O(c^2) \quad [25]$$

for any one of the solids variables denoted here as ψ_s. According to [25], the first-order term is independent of c. A differential equation is then obtained, which, divided by c takes the following form:

$$A_s(r;f)\frac{dV_x}{dr} + B_s(r;V_x)\frac{df}{dr} + C_s(r;f,V_x) = 0. \quad [26]$$

The boundary conditions applying to [26] are given by [22], to which the following should be added:

$$\frac{dV_x}{dr} = 0 \quad \text{for } r = 0, \tag{27}$$

since no additional boundary condition at the wall (such as a no-slip condition) can be formulated for the particles. Again, the boundary conditions are independent of c and the variable coefficients A_s, B_s and C_s are also independent of c *provided that the concentration profiles are self-similar.* Consequently, the solution of [26] with the related boundary conditions is, in turn, independent of the average solids concentration. In other words, *solids velocity profiles independent of average concentration and self-similar concentration profiles are mutually necessary conditions which have to be simultaneously satisfied* otherwise, both are invalid.

In conclusion, the results obtained in this section can be summarized as follows: concentration and velocity profiles of the form

$$\epsilon_s(r) = cf(r; \mathcal{U}), \tag{28}$$

$$V_x(r) = V_x(r; \mathcal{U}) \tag{29}$$

and

$$U_x(r) = U_x^0(r; \mathcal{U}) + \epsilon_s(r; \mathcal{U})U_x^1(r; \mathcal{U}) + O(c^2) \tag{30}$$

are consistent *asymptotic solutions* to the general equations for the dilute phase, fully developed pipe flow of gas–solids suspensions *as the average solids volumetric concentration tends toward zero.* Equations [28]–[30], thus, define *a possible set of flow patterns* in gas–solids suspension flow. The similarity properties of the profiles derived in this approach can be generalized to suspensions flowing in an inclined or horizontal pipe of non-circular cross-section (Molodtsof 1985) provided that dilute phase (i.e. $c \to 0$) and fully developed flow conditions are satisfied.

DISCUSSION

The similar profiles summarized in [28]–[30] are referred to as a *possible* set of flow patterns since the *uniqueness* of these asymptotic solutions cannot be shown using the general equations at the present stage of our knowledge because of the closure problem. It falls, therefore, to experimental investigations to confirm the existence of such a *similar profile regime* in practical cases, for dilute phase flow conditions. In our survey of the literature, we have already observed that several authors reported flow patterns consistent with [28]–[30]. Azzi and co-workers (Azzi 1986; Monceaux *et al.* 1986) reported self-similar particle mass flux profiles for 60 μm cracking catalyst particles carried by atmospheric air in the 144 mm i.d. plexiglass riser of a circulating fluidized bed, for volumetric solids concentrations up to about 4–5%. Consequently, it seems that, the similarity properties of the flow patterns deduced from the theoretical considerations developed above are not a singular solution of the general equations, but correspond to a generalizable feature. Moreover, whereas the definition of what constitutes "dilute phase flow" is still ambiguous in the literature (Mok *et al.* 1989), here it acquires a clear physical significance as it corresponds to the above characterized *similar profile regime.*

The second point to be emphasized concerns the value of the average concentration up to which the similarity properties, or, in other words, the *similar profile regime* is maintained. Again, this limiting value cannot be deduced from theoretical arguments because of the closure problem. It should be noted, however, that, the transition, which is expected to occur, from the *similar profile regime* toward a denser flow regime in which the particle concentration and velocity profiles are no longer self-similar with increasing average concentration, should also be detectable in terms of overall variables, i.e. in a c vs W_s plot. This latter fact can be deduced from [24]. Indeed, as seen in this equation, W_s and c are proportional in the similar profile regime. But as soon as the profiles $f(r)$ and/or $V_x(r)$ begin to deform with increasing solids loading, the proportionality disappears. Consequently, the transition could be seen in the c vs W_s plot as a departure from proportionality.

This indeed occurs, as reported by Azzi and co-workers (Azzi 1986; Monceaux *et al.* 1986). As shown by these authors, and also by the experimental results of Mok *et al.* (1989), beyond transition, the trend of variation of the average concentration with the solids rate is still linear but with a negative intercept. A large number of analogous results can be found in the circulating fluidized bed literature. Moreover, in most cases, the transition results in a sharp change in the linear trend of the variation of c with W_s. As a consequence, the *average relative velocity* is constant in the similar profile regime, while, it increases with increasing concentration beyond the transition. However, the *local relative velocity* which can be deduced from [28] and [29] varies with c even as $c \to 0$.

The third point to which attention should be drawn concerns the fact that the functions on the r.h.s.s of [28]–[30] defining the flow patterns *are unknowns*, since [7], [21] and [26] cannot be solved without solving the closure problem. Consequently, at the present stage of our knowledge, the determination of the profiles falls to experimental investigation. The latter can, however, be guided by the theoretical considerations developed above. The profiles will obviously depend on the gas superficial velocity. A dependence upon gas density, particle density, particle size distribution as well as the pipe diameter is also naturally expected. But the solution of [27] which would give the solids velocity profile, requires the modelling of the s_{rx} term which is determined by the interparticle as well as wall–particle collisions. The effect of these collisions depend upon the shape of individual particles but also on *the roughness and the physical nature of the wall* which are determinant for the momentum transfer during wall–particle collisions. Moreover, as [7], [21] and [27] are intimately coupled by several terms, all the profiles are expected to depend on the same variables. In the experimental determination of the profiles, and thus, also of the correlations involving overall variables, the properties of the wall should be considered among the relevant operating conditions. This latter remark explains, at least partly, the failure of all the "unified" pressure drop correlations reviewed by Muzyka (1985).

CONCLUSIONS

Self-similar concentration and solids velocity profiles have been shown to be solutions to general probabilistic multiphase flow equations under fully developed flow conditions as the average solids concentration tends toward zero. While the theoretical analysis was only able to demonstrate that this result is possible, the experimental evidence available in the literature strongly suggests that similar profiles are a characteristic feature of dilute suspension flows at constant superficial gas velocity.

The analysis further suggests that the deviation from similar profiles as solids concentration is increased can be deduced from relationships between the overall properties (in particular, the plot of the solids rate vs average concentration) of the flow. This too is consistent with available experimental measurements on the transition from dilute to dense flow regimes. Finally, the multiphase flow equation analysis provides a partial explanation for the difficulty associated with developing generalizable suspension pressure drop correlations.

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REFERENCES

ARUNDEL, P. A., BIBB, S. D. & BOOTHROYD, R. G. 1970 Dispersed density distribution and extent of agglomeration in polydisperse fine particle suspension flowing turbulently upwards in a duct. *Powder Technol.* **4**, 302–312.

ARUNDEL, P. A., HOBSON, C. A., LALOR, M. J. & WESTON, W. 1974 Measurements of individual alumina particle velocities and the relative slip of different sized particles in a vertical gas–solid suspension flow using a laser anemometer system. *J. Phys. D: Appl. Phys.* **7**, 2288–2300.

AZZI, M. 1986 Etude des profils de flux de particules dans l'écoulement vertical établi d'une suspension gaz–solides. Thèse de Doctorat, Univ. de Technologie de Compiègne, France.

BIRCHENOUGH, A. & MASON, J. S. 1976 Local particle velocity measurements with a laser anemometer in an upward flowing gas–solid suspension. *Powder Technol.* **14,** 139–152.

DOIG, I. D. & ROPER, G. H. 1967 Air velocity profiles in the presence of cocurrently transported particles. *Ind. Chem. Engng Fundam.* **6,** 247–256.

DRY, R. J. 1987 Radial particle size segregation in a fast fluidized bed. *Powder Technol.* **52,** 7–16.

HARTGE, E. U., LI, Y. & WERTHER, J. 1986 Flow structures in fast fluidized beds. In *Fluidization V* (Edited by ØSTERGAARD, K. & SØRENSEN, A.), pp. 345–352. Engineering Foundation, New York.

HARTGE, E. U., RENSNER, D. & WERTHER, J. 1988 Solids concentration and velocity patterns in circulating fluidized beds. In *Circulating Fluidized Bed Technology II* (Edited by BASU, P. & LARGE, J. F.), pp. 165–180. Pergamon Press, Oxford.

KONNO, H. & SAITO, S. 1969 Pneumatic conveying of solids through straight pipes. *J. Chem. Engng Japan* **2,** 211–217.

KRAMER, T. J. 1970 A study of the mean flow characteristics of gas–solids suspensions flowing in vertical tubes. Ph.D. Thesis, Univ. of Washington, Seattle, Wash.

LEE, S. L. & DURST, F. 1982 On the motion of particles in turbulent duct flows. *Int. J. Multiphase Flow* **8,** 125–146.

MAEDA, M., HISIDA, K. & FURUTANI, T. 1980 Optical measurements of local gas and particle velocity in an upward flowing dilute gas–solids suspension. In *Proc. Polyphase Flow and Transport Technology (Century 2-ETC, San Francisco, Calif.)*, pp. 211–216.

MOK, S. L. K., MOLODTSOF, Y., LARGE, J. F. & BERGOUGNOU, M. A. 1989 Characterization of dilute and dense phase vertical upflow gas–solids transport based on average concentration and velocity data. *Can. J. Chem. Engng* **67,** 10–16.

MOLODTSOF, Y. 1985 Equations générales probabilistes des écoulements polyphasiques et applications aux mélanges gaz–solides. Thèse de Doctorat d'Etat, Univ. de Technologie de Compiègne, France.

MOLODTSOF, Y. & MUZYKA, D. W. 1989 General probabilistic multiphase flow equations for analyzing gas–solids mixtures. *Int. J. Engng Fluid Mech.* **2,** 1–24.

MONCEAUX, L., AZZI, M., MOLODTSOF, Y. & LARGE, J. F. 1986 Particle mass flux profiles and flow regime characterization in a pilot-scale fast fluidized bed unit. In *Fluidization V* (Edited by ØSTERGAARD, K. & SØRENSEN, A.), pp. 337–344. Engineering Foundation, New York.

MUZYKA, D. W. 1985 The use of probabilistic multiphase flow equations in the study of the hydrodynamics and heat transfer in gas–solids suspensions. Ph.D. Thesis, Univ. of Western Ontario, London, Ontario.

MUZYKA, D. W., MOLODTSOF, Y., LARGE, J. F. & BERGOUGNOU, M. A. 1983 Applications of probabilistic multiphase flow equations to dilute phase pneumatic transport. Presented at the *33rd Canadian Chemical Engineering Conf.*, Toronto, Ontario.

REDDY, K. V. S. & PEI, D. C. T. 1969 Particle dynamics in gas–solids flow in a vertical pipe. *Ind. Chem. Engng Fundam.* **8,** 490–497.

RHODES, M. J., LAUSSMANN, P., VILLAIN, F. & GELDART, D. 1988 Measurements of radial and axial solids flux variations in the riser of a circulating fluidized bed. In *Circulating Fluidized Bed Technology II* (Edited by BASU, P. & LARGE, J. F.), pp. 155–164. Pergamon Press, Oxford.

TSUJI, Y., MORIKAWA, Y. & SIOMI, H. 1984 LDV measurements of an air–solid two-phase flow in a vertical pipe. *J. Fluid Mech.* **139,** 417–434.

WEINSTEIN, H., SHAO, M., SCHNITZLEIN, M. & GRAFF, R. A. 1986 Radial variation in void fraction in a fast fluidized bed. In *Fluidization V* (Edited by ØSTERGAARD, K. & SØRENSEN, A.), pp. 329–336. Engineering Foundation, New York.

VAN ZOONEN, D. 1962 Measurements of diffusional phenomena and velocity profiles in a vertical riser. In *Proc. Symp. on Interaction Between Fluids and Particles*, Inst. Chem. Engrs, London, pp. 64–71.

APPENDIX

In the probabilistic Eulerian approach used by Molodtsof & Muzyka (1989) to derive the general equations, mean local variables of a phase are defined using the "immediate" (i.e. actual and random) Eulerian variables of that phase. This definition is based on the use of the phase

characteristic functions such as $\hat{K}_p(x_i, t)$, which takes on the value of 1 when the point whose coordinates are x_i is located within phase p and is equal to 0 in all other cases. $\hat{K}_p$ is a random variable as indicated by the $\hat{}$ superscript. The probability of the presence of phase p is defined as the probabilistic mean (or expected value) of $\hat{K}_p$:

$$\epsilon_p(x_i, t) = \overline{\hat{K}_p}(x_i, t), \qquad [A.1]$$

where the overbar indicates the ensemble averaging operator. Although $\hat{K}_p$ is a piecewise continuous function of space coordinates and time, ϵ_p is defined throughout the mixture and can be considered in most applications, as a continuously differentiable function.

When a physical variable $\hat{\psi}_p$ associated with phase p is under consideration, its associated probabilistic mean ψ_p is defined as follows:

$$\epsilon_p \psi_p = \overline{\hat{K}_p \hat{\psi}_p}. \qquad [A.2]$$

Hence, for the phase mean value τ_{rx} of the fluid viscous stress component $\hat{\tau}_{rx}$ one has

$$\epsilon_f \tau_{rx} = \overline{\hat{K}_f \hat{\tau}_{rx}}. \qquad [A.3]$$

For an incompressible Newtonian fluid $\hat{\tau}_{rx}$ is related to the random fluid velocity components by the following constitutive equation:

$$\hat{\tau}_{rx} = \mu\left(\frac{\partial \hat{U}_r}{\partial x} + \frac{\partial \hat{U}_x}{\partial r}\right). \qquad [A.4]$$

Consequently, substituting [A.4] in [A.3], and considering $\hat{K}_f$ as a distribution (or generalized function) which corresponds to its real nature, one obtains:

$$\epsilon_f \tau_{rx} = \mu\left[\overline{\frac{\partial}{\partial x}(\hat{K}_f \hat{U}_r)} + \overline{\frac{\partial}{\partial r}(\hat{K}_f \hat{U}_x)}\right] - \mu\left[\overline{\hat{U}_r \frac{\partial \hat{K}_f}{\partial x}} + \overline{\hat{U}_x \frac{\partial \hat{K}_f}{\partial r}}\right]. \qquad [A.5]$$

The derivatives of the characteristic function with respect to space coordinates are the components of a vectorial delta distribution non-zero at the interface. The probabilistic mean of the product of a random physical variable $\hat{\psi}_p$ by such a derivative, therefore, corresponds to an average of the values taken on by the variable along the interface (Molodtsof & Muzyka 1989). In order to be consistent with definitions such as [A.2], the "interfacial mean" $\psi_p^!$ of $\hat{\psi}_p$ is defined in the form:

$$\psi_p^! \frac{\partial \epsilon_p}{\partial x_i} = \overline{\hat{\psi}_p \frac{\partial \hat{K}_p}{\partial x_i}}. \qquad [A.6]$$

When [A.2] and [A.6] written for the fluid velocity components are substituted in [A.5], one obtains:

$$\epsilon_f \tau_{rx} = \mu\left[\frac{\partial}{\partial x}(\epsilon_f U_r) + \frac{\partial}{\partial r}(\epsilon_f U_x)\right] - \mu\left(U_r^! \frac{\partial \epsilon_f}{\partial x} + U_x^! \frac{\partial \epsilon_f}{\partial r}\right). \qquad [A.7]$$

In fully developed flow the axial derivatives of the phase presence probabilities are zero as well as the radial component of the fluid velocity. Therefore, in [A.7] the first and third terms on the r.h.s. vanish identically and the equation reduces to

$$\epsilon_f \tau_{rx} = \mu \frac{\partial}{\partial r}(\epsilon_f U_r) - \mu U_x^! \frac{\partial \epsilon_f}{\partial r}. \qquad [A.8]$$

Along the fluid–particle interface, the jump condition for the velocity components is the no-slip condition. Moreover, the derivatives of the characteristic functions of the fluid and solids phases are equal with the opposite sign, all along the interface. Consequently, according to [4], one has

$$U_x^! \frac{\partial \epsilon_f}{\partial r} = -V_I \frac{\partial \epsilon_s}{\partial r}, \qquad [A.9]$$

where V_I is the axial component of the "interfacial mean" solids velocity. Substituting [A.9] in [A.8] then gives

$$\epsilon_f \tau_{rx} = \mu\left[\frac{\partial}{\partial r}(\epsilon_f U_r) + V_I \frac{\partial \epsilon_s}{\partial r}\right]. \qquad [A.10]$$